

# TONBRIDGE SCHOOL

Scholarship Examination 2010

## MATHEMATICS I

Tuesday 27th April 2010  
11.15 am

Time allowed: 1 hour 30 minutes

*Answer as many questions as you can.  
Questions 1 to 5 are worth 8 marks each;  
Questions 6 to 9 are worth 15 marks each.*

*All answers must be supported by adequate explanation.  
Calculators may be used in any question.*

$$\frac{x+1}{2} = y+6, \quad \frac{x-2}{3} = \frac{2-y}{5}$$

[8 marks]

2. At precisely what time between 12 o'clock and 1 o'clock is the clockwise angle from the hour hand to the minute hand of a clock equal to  $231^\circ$ ? [8]

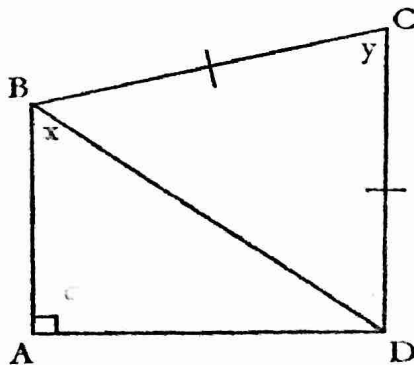
3. In this question,  $X$  is the number with prime factorisation  $2^4 \times 11$ . A *square number* is a member of the sequence 1, 4, 9, 16, ...; a *cube number* is a member of the sequence 1, 8, 27, 64, ...

- (a) Explain why  $11X$  is a square number and give its square root as a product of prime numbers.
- (b) What is the smallest number which, when multiplied by  $X$ , gives a cube number?
- (c) Find a number by which  $X$  must be multiplied to make it both a cube number and a square number. [8]

4. In suitable units, the power  $P$  consumed by a current  $I$  passing through a wire of length  $L$  and diameter  $D$  is given by the formula  $P = \frac{4I^2L}{D^2}$ .

- (a) If  $I = 2.3$ ,  $L = 15.6$ ,  $D = 0.7$ , find the value of  $P$ .
- (b) If  $P = 40.8$ ,  $I = 3.5$ ,  $D = 1.2$ , find the value of  $L$ .
- (c) If  $P = 7.5$ ,  $D = 1.4$ ,  $L = 4.6$ , find the value of  $I$ .
- (d) If  $P = 79.1$ ,  $I = 4.8$ ,  $L = 10.9$ , find the value of  $D$ . [8]

5. In the figure, triangle ABD has a right-angle at A and triangle BCD is isosceles with  $BC = CD$ . The line BD bisects the angle ADC, so that angle ADB is equal to angle BDC. Angle ABD is  $x$  and angle BCD is  $y$ .

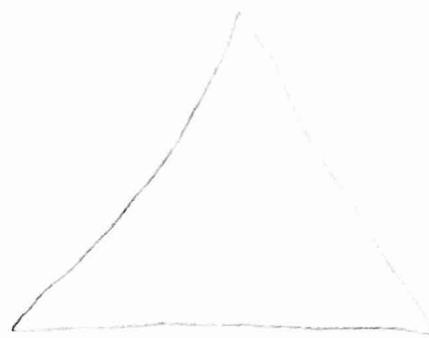
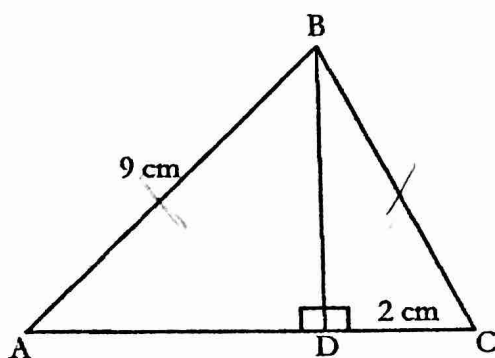


- (a) (i) If  $x = 60^\circ$ , find  $y$ .  
(ii) If  $y = 25^\circ$ , find  $x$ .

- (b) On the basis of your answers in (a), what is the probable relationship between  $x$  and  $y$ ?
- (c) Use algebra to show that the relationship in (b) holds whatever the value of  $x$ .

[8]

6. In the figure,  $D$  is on  $AC$  with  $BD$  perpendicular to  $AC$ ,  $AB = 9\text{ cm}$  and  $DC = 2\text{ cm}$ . You are also given that triangle  $ABC$  is isosceles.



- (a) By sketching the triangles, show there are three different triangles  $ABC$  that satisfy these conditions.
- (b) Find both the area and the perimeter of any TWO of the triangles  $ABC$  in (a).

[15]

7. The height ( $y$  cm) of a solid cylinder of radius  $x$  cm with total surface area  $320\text{ cm}^2$  is given by the formula  $y = \frac{160}{\pi x} - x$ , where  $\pi = 3.14159\dots$  has its usual meaning.

- (a) When  $x = 4$ , show that  $y = 8.73$  (correct to 2 decimal places).
- (b) Find the values of  $y$  for  $x = 1.5, 2, 3, 5, 6, 8$ .
- (c) Choosing sensible scales, use your values to plot a graph of  $y$  against  $x$ .
- (d) Comment on the value of  $y$  when  $x = 8$ . *Surplus min possible value of  $y$ .*
- (e) Use your graph to find the value of  $x$  that gives a height of 20 cm.
- (f) Read off the value of  $x$  when  $y = 0$ . What has happened to the cylinder in this situation?

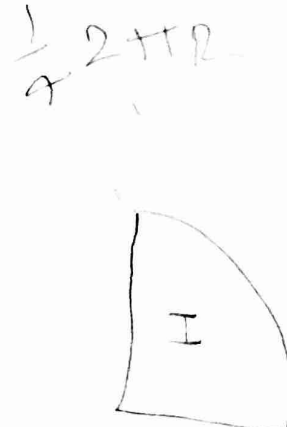
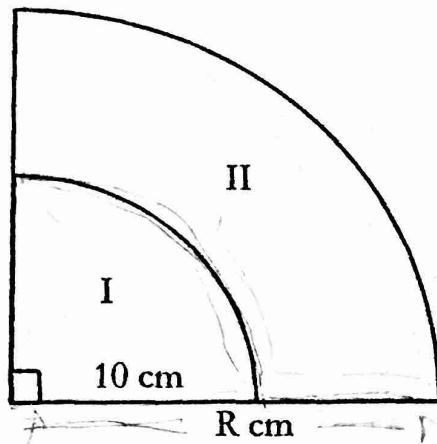
[15]

Maximum

$$y = \frac{160}{\pi(4)} - 4$$



8. The figure below shows two quadrants of a circle. The smaller one has radius 10 cm, the larger one has radius  $R$  cm. The larger quadrant is divided into two regions. Region I is the smaller quadrant; Region II is outside the smaller quadrant but inside the larger quadrant.



- (a) Find  $R$  if Regions I and II have the same area.  
 (b) Find  $R$  if Regions I and II have the same perimeter.

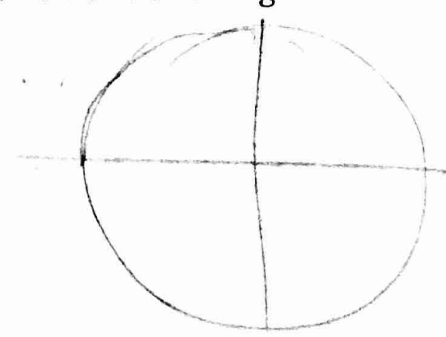
[15]

9. Look carefully at the pattern of numbers in the table below.

$\frac{\pi r^2}{4}$

Row 1	$1 = 0 + 1$
Row 2	$2 + 3 + 4 = 1 + 8$
Row 3	$5 + 6 + 7 + 8 + 9 = 8 + 27$
Row 4	
Row $n$	$A + \dots + B = C + D$

- (a) Write down Row 4, following the pattern in the table.  
 (b) In any row (such as the bottom row in the table), the sum on the left-hand side begins with the number  $A$  and ends with the number  $B$ ; the two numbers on the right-hand side are  $C$  and  $D$ .  
 (i) If  $B = 841$ , find the values of  $A, C, D$ .  
 (ii) If  $A = 1090$ , find the values of  $B, C, D$ .  
 (iii) If  $C = 3375$ , find the values of  $A, B, D$ .  
 (iv) If  $D = 59319$ , find the values of  $A, B, C$ .  
 (c) For Row  $n$ , write down formulae for each of  $A, B, C, D$  in terms of  $n$ .



[15]